

Broadcasting a message over erasure channels with cooperating receivers

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Abstract—In this paper we consider the orthogonal cooperative broadcast channel with a common message. The downlink and cooperation channels are modelled by erasure channels and assumed to be orthogonal. We derive an achievable rate for this channel by using a combination of decode-and-forward and estimate-and-forward and considering both symmetric and asymmetric cooperation. These schemes are based on a two-step cooperation. It is shown under which conditions the proposed achievable rate coincides with the considered upper bound that is in fact the interpretation of the traditional cut-set bound in the context of broadcast cooperative erasure channels. Moreover we propose a time-sharing-based iterative coding scheme with more than two cooperation steps.

I. INTRODUCTION

In this paper we derive the achievable rate of a cooperative broadcast channel (CBC) with one transmitter and two receivers. The channel under consideration has essentially four main features:

- The receivers are interested in the same message *i.e.* the transmitter does not send any private message. This situation is commonly referred to as the single common message case.
- The receivers can cooperate in order to enhance the transmission data rate.
- The different links between the transmitter and receivers and between the receivers themselves are erasure channels. In this setting symbols sent over the channel are received errorless or erased and replaced by an erasure symbols $\{e\}$. These channels are very relevant for modeling the packet layer transmission in which each packet can be viewed as a symbol lying in a large alphabet.
- The downlink and cooperation channels do not interfere (orthogonality assumption). Designing a relay-receiver that receives and transmits in the same bandwidth at the same time is not an easy task. This is one of the reasons why the orthogonality assumption makes sense.

To the author's knowledge the technical background of this problem reduces to a few papers. The (general) discrete broadcast channel with a bidirectional conference link and a single common message was originally studied by Draper et al. in [2]. The authors proposed a way of decoding the message in multiple rounds. The coding-decoding scheme is based on the use of auxiliary variables while a certain form of

channel comparability is assumed through these variables¹.

This channel has also been analyzed by [6] where the authors essentially proposed achievable rates based on the use of estimate-and-forward (EF) at both receivers. The Gaussian counterpart of this channel has been studied in [7]. Showing the optimality of decode-and-forward (DF) for a unidirectional cooperation the authors evaluated the exact loss due to orthogonalization of the cooperation channel. For the bidirectional case, the proposed achievable rate is based on a combination of EF and DF and shown to always outperform the pure EF-based solution. Independently [8] exploited a similar approach to analyze the Gaussian relay channel with a bidirectional cooperation. As for the fading case it has been partially treated in [9]. The diversity-multiplexing trade-off, achieved by using a "dynamic" version of DF, is derived for the unidirectional cooperation case.

Apart from [2] where the idea of multi-step decoding is illustrated by the binary erasure channel none of the aforementioned works tackled the erasure CBC directly. Regarding this point the closest work to what is done in this paper is that of [10], which analyzes the erasure relay channel. The main result of [10] is that the channel capacity is determined whereas it is still unknown for the general discrete relay channel. This explain why we investigated the channel considered in this paper.

As a summery, in this paper we focus on the case where all the channels are erasure channels, which means, in particular, that the cooperation link is not a conference link but consists of two erasure channels. We first propose a 1-step (symmetric cooperation) and 2-step (asymmetric cooperation) coding/decoding schemes based on a combination of DF and EF. We then show the performance of applying a multi-step coding/decoding schemes based on time-sharing in the receiver side. Note that, apart from the fact that no auxiliary random variables and form of channel comparability is assumed, the proposed schemes also differ from [2] because we use deterministic coding schemes instead of using random coding schemes. In general the proposed rates do not coincide with the considered upper bound but the gap is shown to be small for practical ranges of erasure parameters and even zero

¹Commenting on this concept is out of the scope of this paper. For more information see [3], [4], [5]. Example: The channel $p(y_1|x)$ is said to be less noisy than $p(y_2|x)$ if for any auxiliary random variable U , $I(U;Y_1) \geq I(U;Y_2)$.

in the not so restrictive high cooperation regime. Before providing the corresponding analysis (section III) we first review a few important recent results [10] that are needed to understand this analysis (section II). At the end of the paper we provide possible extensions of this work.

II. THE ERASURE RELAY CHANNEL

The main purpose of this section is to understand how the two famous relaying protocols decode-and-forward and estimate-and-forward work in the context of erasure channels. Before providing the corresponding description we review the point-to-point erasure channel, the relay erasure channel definition and an upper bound for its capacity.

A *point-to-point erasure channel* is defined by an input alphabet \mathcal{X} , an output alphabet \mathcal{Y} and a conditional distribution function $\mathcal{E}_k(\mathbf{e}|x_k)$ which is the probability that an erasure is observed at the output of the channel given that a symbol x_k was sent over the channel at time k . We consider that erasure events do not depend on transmitted symbols, *i.e.* the erasure channel is only characterized by its erasure probability p_k . As the channel is also assumed to be memoryless the superscript k will be dropped from the erasure parameter. The capacity of this channel is merely given by $C = 1 - p$, *e.g.* [1], and can be achieved by using MDS (maximum separable distance) codes. This comes from the fact that the minimum distance (say d_{\min}) of MDS codes coincides with the Singleton bound and the decoder can detect $d_{\min} - 1$ errors, which leads to the capacity for infinite data block length (see [10] for more details).

The *orthogonal relay erasure channel* ($\mathcal{X} \times \mathcal{X}_{12}, p(y_1, y_2, y_{12}|x, x_{12}), \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_{12}$) consists of five finite alphabets and a collection of probability distributions $p(\cdot, \cdot, \cdot | x, x_{12})$ on $\mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_{12}$, one for each $(x, x_{12}) \in \mathcal{X} \times \mathcal{X}_{12}$. The channel is orthogonal in the sense that $p(y_1, y_2, y_{12}|x, x_{12}) = p(y_1, y_2|x)p(y_{12}|x_{12})$, which means that the two signal received by the destination do not "interfere" each other. Moreover, erasures are assumed to be spatially independent in such a way that each erasure channel is characterized by its erasure probability. This channel can be therefore characterized by erasures parameters: p_1 , p_2 , and p_{12} which are respectively the average erasure probability over the sender-relay, sender-destination, and relay-destination channels. (see figure 1).

The capacity of the orthogonal erasure channel can be upper bounded according to the following theorem firstly proposed in [10].

Theorem 2.1 (Upper bound for the relay erasure channel): The rate R achieved by any deterministic coding schemes over the relay erasure channel is necessarily subject to the following inequalities:

$$\begin{cases} R < 1 - p_1 p_2 & \text{if } p_1 \geq p_{12} \\ R < \max\{T, 1 - p_2\} & \text{if } p_1 < p_{12} \end{cases}$$

with $T = \min\{1 - p_1, 1 - p_2 + 1 - p_{12}\}$

□

This bound can be attained by using a linear deterministic coding scheme with a complexity $\mathcal{O}(n \log(n))$ where n is

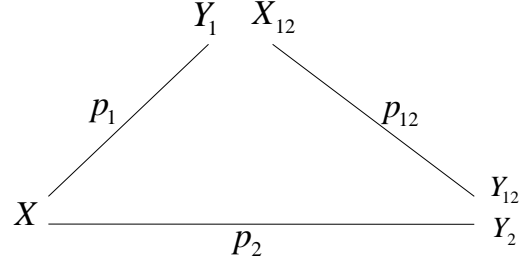


Fig. 1. Single relay erasure channel

the codeword length. Almost-MDS codes [11] with linear decoding complexity are also applicable in this context. It is worthy to say that the underlying capacity-achieving codes do not rely on any type of side information, only the knowledge of the average packet losses over the transmission links is needed. As illustrated previously in [10], if $p_1 \geq p_{12}$ the optimal strategy is to apply estimate-and-forward and the achievability region is bounded by $1 - p_1 p_2$, while if $p_1 < p_{12}$ no one can do better than applying a decode-and-forward scheme with the achievable rate T . Let us describe in what these protocols consists of for the erasure relay channel.

A. Decode-and-forward

Assume an (n, nR) MDS code with generator matrix $\mathbf{G} = [\mathbf{I}_{nR \times nR} | \mathbf{A}_{nR \times (n-nR)} | \mathbf{B}_{nR \times \ell}]$. At the transmitter the message symbols are encoded through the coding matrix $\mathbf{G}_t = [\mathbf{I}_{nR \times nR} | \mathbf{A}_{nR \times (n-nR)}]$ that is to say that $(n - nR)$ redundant packets are introduced. Now we assume that $R < 1 - p_1$, which allows us to insure reliable decoding at the relay. The relay re-encodes the (perfectly) decoded message symbols by using the coding matrix $\mathbf{B}_{nR \times \ell}$.

For n large the destination receives about $n \times (1 - p_2)$ reliable packets from the source and about $\ell \times (1 - p_{12})$ reliable packets from the relay. Therefore the receiver can reliably decode an MDS code with generator matrix \mathbf{G} provided that $nR < n(1 - p_2) + \ell(1 - p_{12})$. The rate $1 - p_2 + \alpha \times (1 - p_{12})$ is then achievable for any $\alpha \in [0, 1]$, where $\alpha \triangleq \frac{\ell}{n}$ stands for the relay node sharing. Note that for $\alpha \rightarrow 0$ the relay is not used and the achieved rate is merely $1 - p_1$.

B. Estimate-and-forward

Assume an (n, nR) MDS code with generator matrix $\mathbf{\Gamma} = [\mathbf{I}_{nR \times nR} | \mathbf{C}_{nR \times (n-nR)}]$. As we assume the erasure events at the relay and receiver to be independent (spatial independency assumption) the are about $n_e = n(p_1 + p_2 - p_1 p_2)$ erased packets in total, with n large. Said otherwise the relay and receiver have $n - n_e = n(1 - p_1)(1 - p_2)$ reliable packets in common. So the $n(1 - p_2)$ reliable packets at the receiver can be partitioned into two sets: That the relay already has $(n(1 - p_1)(1 - p_2))$ packets and that the relay does not have $(n(1 - p_2)p_1)$ packets. In the same way the $n(1 - p_1)$ reliable packets at the relay can be partitioned into two sets: That the receiver already has $(n(1 - p_1)(1 - p_2))$ packets

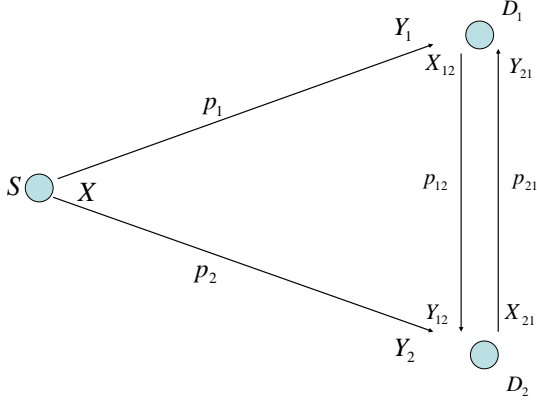


Fig. 2. Erasure cooperative broadcast channel

and that the receiver does not have $(n(1-p_1)p_2)$ packets). In the ideal situation the relay would be able to send the $n(1-p_1)p_2$ missing packets to the receiver. However this is not possible because the relay does not know which packets the receiver failed to decode reliably. As this side information is assumed to be unavailable at the relay, a possible solution to this problem is that the relay re-encodes all the reliably received packets. This is the idea of estimate-and-forward in the context of erasure channels.

To implement EF the relay uses an MDS code to re-encode the $k = n(1-p_1)$ reliable packets he received from the source by adding $n-k$ redundant packets. The k extra packets will be decoded reliably by the receiver provided that $p_{12} < p_1$. Out of these packets there are kp_2 reliable packets that have not been received by the destination through the direct link. The destination has $n(1-p_2) + kp_2$ reliable packets. Therefore the receiver can perfectly decode the MDS code provided that $nR < n(1-p_2) + kp_2$. This shows that the rate $1 - p_1p_2$ is achievable.

III. THE ERASURE BROADCAST CHANNEL WITH COOPERATING RECEIVERS

Now we consider the channel under investigation in this paper that is the erasure cooperative broadcast channel with a single common message. This channel differs from the relay channel for essentially two reasons. First: For the relay channel only the receiver has to decode the transmitted message reliably whereas in the erasure CBC both "relay-receiver" nodes have to decode the transmitted message. Second: In contrast with the conventional relay channel, the cooperation link is bi-directional. Now let us define the erasure CBC properly.

The *orthogonal erasure cooperative broadcast channel* $(\mathcal{X} \times \mathcal{X}_{12} \times \mathcal{X}_{21}, p(y_1, y_2, y_{12}, y_{21} | x, x_{12}, x_{21}), \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_{12} \times \mathcal{Y}_{21})$ consists of six finite alphabets and a collection of probability distributions $p(\cdot, \cdot, \cdot, \cdot | x, x_{12}, x_{21})$ on $\mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_{12} \times \mathcal{Y}_{21}$, one for each $(x, x_{12}, x_{21}) \in \mathcal{X} \times \mathcal{X}_{12} \times \mathcal{X}_{21}$. The channel is orthogonal in the sense that $p(y_1, y_2, y_{12}, y_{21} | x, x_{12}, x_{21}) = p(y_1, y_2 | x)p(y_{12} | x_{12})p(y_{21} | x_{21})$, which means that the two signal received by a receiver

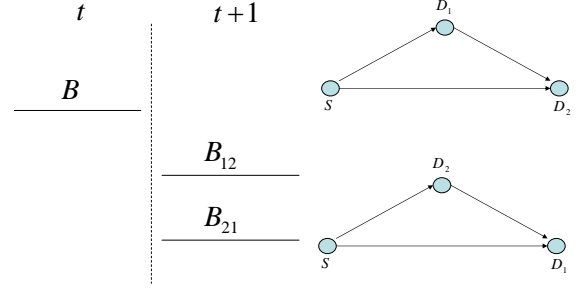


Fig. 3. Symmetric cooperation schemes for the erasure CBC

do not "interfere" each other. As for the relay channel we assume erasures to be spatially independent in such a way that each erasure channel is characterized by its erasure probability. The overall channel is then characterized by four erasures parameters: p_1, p_2, p_{12}, p_{21} (see figure 2).

As we did for the relay channel we now provide an upper bound for the coding rate (say R) over this channel. To this end we use the cut-set bound derived by [6] and replace the conference link with noisy point-to-point channels. The general discrete cut-set bound is then given by:

$$R \leq \sup_{p(x, x_{12}, x_{21})} \min \{ I(X; Y_1) + I(X_{21}; Y_{21}), I(X; Y_2) + I(X_{12}; Y_{12}), I(X; Y_1, Y_2) \}.$$

As mentioned in [7] this bound is tight in at least two special cases of the CBC: The unidirectional case ($X_{21} \equiv \emptyset$) and the deterministic case ($Y_1 = f_1(X), Y_2 = f_2(X)$).

By applying the Shearer theorem (see [10]) to this bound it can be shown to become:

$$R \leq \min \{ 1 - p_1 + 1 - p_{21}, 1 - p_2 + 1 - p_{12}, 1 - p_1p_2 \}. \quad (1)$$

Now we turn our attention to the rates achievable for this channel. For sake of clarity we assume without loss of generality that orthogonality is implemented by frequency division: The downlink and cooperation channels have non-overlapping bands of frequency say B_{dl} and B_{coop} . To implement orthogonality between the two channels $X_{12} \rightarrow Y_{12}$ and $X_{21} \rightarrow Y_{21}$ one has to split the cooperation band of frequency in at least two sub-bands, say B_{12} and B_{21} . The overall bandwidth constraint imposes that $B_{dl} + B_{12} + B_{21} = B$. More generally one could split the cooperation band into K orthogonal sub-channels but in the current version of the paper we limited ourselves to the minimum value of K to implement the channel under consideration which is 2. Under this assumption there are three ways of cooperating as figure 3 and 4 show. Now let us provide the rate achieved by the three considered coding schemes.

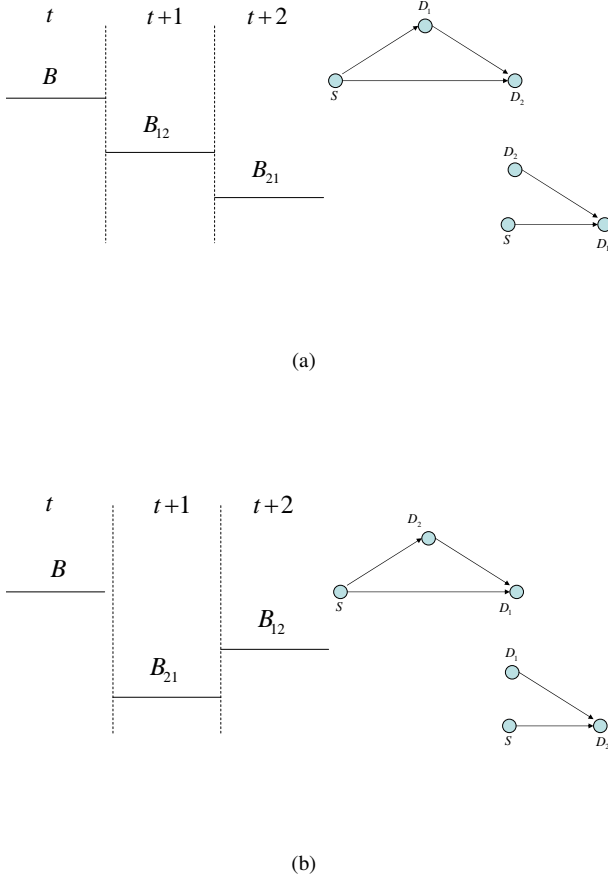


Fig. 4. Asymmetric cooperation schemes for the erasure CBC

A. Coding scheme "a"

For the first coding scheme the cooperation channel is used in a symmetric way. The transmitter send a block of data and the relay-receivers re-encode this block by using either decode-and-forward (when $p_i < p_{ij}$ for $i \neq j$) or estimate-and-forward (when $p_i \geq p_{ij}$ for $i \neq j$). The relay-receivers send these re-encoded blocks to each other *simultaneously*. The receivers can then decode the transmitted message in two blocks. By using this coding scheme the channel can be decomposed into a superposition of two relay channels illustrated in figure 3. The transmitter encodes the message into a codeword of length n by using the generator matrix $\mathbf{G}_a = [\mathbf{I}_{nR_a \times nR_a} | \mathbf{A}_{nR_a \times (n-nR_a)}]$. For $i \in \{1, 2\}$ the relay-receiver uses the generator matrix $\mathbf{B}_{nR_a \times n}^{(i)}$. Therefore the following transmission rate can be achieved:

$$R_a = \min \{R_a^{(1)}, R_b^{(2)}\}$$

with

$$R_a^{(1)} = \begin{cases} 1 - p_1 p_2, & \text{if } p_1 \geq p_{12} \\ \max\{T_a^{(1)}, 1 - p_2\} & \text{if } p_1 < p_{12} \end{cases}$$

and

$$R_a^{(2)} = \begin{cases} 1 - p_1 p_2, & \text{if } p_2 \geq p_{21} \\ \max\{T_a^{(2)}, 1 - p_1\} & \text{if } p_2 < p_{21} \end{cases}$$

with $T_a^{(1)} = \min\{1 - p_1, 1 - p_2 + 1 - p_{12}\}$ and $T_a^{(2)} = \min\{1 - p_2, 1 - p_1 + 1 - p_{21}\}$.

B. Coding scheme "b"

For the previous coding scheme both receivers can decode within two transmission blocks. Here we consider an asymmetric way of cooperating. This time the receivers do not relay in the same time (see figure 4(a)). Let t be the block or time index. At time t the transmitter sends the data block. At time $t + 1$ receiver 1 applies the relaying DF/EF scheme and send the corresponding block to receiver 2. During this time receiver 2 waits for his partner. At time $t + 2$ receiver 2 applies the relaying scheme DF/EF from his knowledge (the data block from the source + the data block from receiver 1). So receiver 2 has to decode within two blocks and receiver 1 has to decode within three blocks. Compared to the coding scheme "a" we see that one of the receiver can take advantage of an enhanced decoding whereas the other one does not improve its performance with respect to that obtained with coding scheme "a". Let us translate the proposed cooperation scheme into achievable rates.

For insuring reliable decoding at receiver 2 the transmission rate cannot exceed the following value:

$$R_b^{(2)} = \begin{cases} 1 - p_1 p_2, & \text{if } p_1 \geq p_{12} \\ \max\{T_b^{(2)}, 1 - p_2\} & \text{if } p_1 < p_{12} \end{cases}$$

where $T_b^{(2)} = T_a^{(2)}$. Now consider receiver 1; for n large it receives about $n(1 - p_1)$ reliable packets from the source and $n(1 - p_{21})$ reliable packets from receiver 2. Always by using an MDS code receiver 1 can reliably decode the received packets if the transmission rate verifies: $n(1 - p_1) + n(1 - p_{21}) > nR_b^{(1)}$. The rate achieved by receiver 2 is:

$$R_b^{(1)} = 1 - p_1 + 1 - p_{21}.$$

Eventually the transmission rate that can be achieved is the minimum of the rates derived *i.e.* :

$$R_b = \min \{R_b^{(1)}, R_b^{(2)}\}.$$

C. Coding scheme "c"

For coding scheme "b" we assume that receiver 1 relays first while receiver 2 waits for the block duration # $t + 1$. Obviously we can reverse the roles of two receivers (figure 4(b)). In practice this could mean for example that the "best receiver" starts the cooperation. One obtains the following achievable rate:

$$R_c = \min \{R_c^{(1)}, R_c^{(2)}\}.$$

with

$$R_c^{(1)} = \begin{cases} 1 - p_1 p_2, & \text{if } p_2 \geq p_{21} \\ \max\{T_c^{(2)}, 1 - p_1\} & \text{if } p_2 < p_{21} \end{cases}$$

and

$$R_c^{(2)} = 1 - p_2 + 1 - p_{12}.$$

As one can always choose the best cooperation scheme among the three coding schemes under consideration, the resulting achievable transmission rate is finally given by the following inequality: $R \leq \max\{R_a, R_b, R_c\}$.

Now let us denote by \mathcal{C}_1 the hyper-cube $\{(x, y, z, t) \in \mathbb{R}^4 : 0 \leq x, y, z, t \leq 1\}$ and define the following domain:

$$\mathcal{D} = \left\{ (p_1, p_2, p_{12}, p_{21}) \in \mathcal{C} : \begin{cases} p_1 + p_{21} - 1 < p_2 < p_{21} \\ p_2 + p_{12} - 1 < p_1 < p_{12} \end{cases} \right\}.$$

By using these notations the rate achieved by the proposed cooperation scheme can be rewritten as:

$$R = \begin{cases} \max\{1 - p_1, 1 - p_2\} & \text{if } (p_1, p_2, p_{12}, p_{21}) \in \mathcal{D} \\ \min\{1 - p_1 + 1 - p_{21}, \\ 1 - p_2 + 1 - p_{12}, \\ 1 - p_1 p_2\} & \text{if } (p_1, p_2, p_{12}, p_{21}) \in \mathcal{C} \cap \overline{\mathcal{D}}. \end{cases}$$

Clearly the proposed achievable rate coincides with the upper bound 1 for $(p_1, p_2, p_{12}, p_{21}) \in \mathcal{C} \cap \mathcal{D}$. The question is then: How "big" is \mathcal{D} ? First note that in real situations the erasure probabilities are less than 50 %, which amounts to describing \mathcal{D} by only two inequalities: $p_2 < p_{21}$ and $p_2 < p_{12}$. This means that the proposed scheme is not capacity-achieving when the two cooperation channels are "worse" than their corresponding downlink channels.

So far we limited ourselves to the case where cooperation is performed into two step. Now we propose a coding-decoding that allows for more cooperation steps following the original idea of [2] ($M > 2$). For this purpose a new way of implementing decode-and-forward will be used.

D. Iterative coding scheme

Assume an (n, nR) MDS code with generator matrix $\mathbf{G}_t = [\mathbf{I}_{nR \times nR} | \mathbf{A}_{nR \times (n-nR)}]$ at the transmitter. The n -length codewords are broadcasted over the channel and as the erasure patterns over the downlink channels are independent the maximum number of packets the "best" receiver can decode reliably equals $n(1 - p_1 p_2)$. The two receivers decode $n(1 - p_1)(1 - p_2)$ packets in common while receiver 1 (resp. receiver 2) decodes $np_2(1 - p_1)$ packets (resp. $np_1(1 - p_2)$). Before going further let us introduce the following notations: \mathcal{D}_1 will stand for the set of packets decoded by receiver 1, \mathcal{D}_2 will stand for the set of packets decoded by receiver 2 and \mathcal{D}_c will stand for the set of common decoded packets. The ideal cooperation situation would be that receiver 2 (resp. receiver 1) receives all the packets in the set $\mathcal{D}_2 \cap \mathcal{D}_c$ (resp. $\mathcal{D}_1 \cap \mathcal{D}_c$). This ideal situation assumes a certain form of side information that was not available for the coding-decoding schemes "a", "b" and "c". Instead if assuming this side information, which would not be fair, we will exploit the averaging effect of the erasures that can be obtained thanks to random coding.

Let's assume that D_1 is the receiver who starts the conversation. At the first round, D_1 chooses randomly k_{11} packets from \mathcal{D}_1 and encodes them through an MDS code of rate $R_{12} = 1 - p_{12}$ and transmits the resulting encoded packets over the cooperation channel. These k_{11} packets will be reliably decoded by D_2 provided that $R_{12} \leq (1 - p_{12})$. A fraction of these packets, equal to $1 - p_1$, is in \mathcal{D}_c and therefore useless for the final decoding process at D_2 ; however, because D_2 is now aware of them it will not select them anymore to be transferred to D_1 . Thus they can contribute to improving the cooperation process by providing a type of side information in D_2 . At the second round, D_2 selects randomly k_{22} packets from \mathcal{D}_2 that are not in the set of duplicated packets received from D_1 . These packets will be encoded through an MDS code of rate $R_{21} = 1 - p_{21}$ and the resulting encoded packets are transmitted over the cooperation channel. D_1 will reliably decode the selected k_{22} packets where $\alpha_{22} = \frac{|\mathcal{D}_c| - k_{11}(1 - p_1)}{|\mathcal{D}_2| - k_{11}(1 - p_1)}$ 100 percent of them has been already received at D_1 through the sender. The encoding process in the next rounds is straightforward; for example at the third round D_1 will choose randomly a subset of packets in \mathcal{D}_1 (with size k_{13}) that has no intersection with the set of duplicated packets received from D_2 in the previous rounds (in this case the second round of cooperation) and none of them has been yet sent by D_1 to D_2 . The selected packets will be decoded and sent over the cooperation channel to D_2 where $\alpha_{13} = \frac{|\mathcal{D}_c| - k_{11}(1 - p_1) - k_{22}\alpha_{22}}{|\mathcal{D}_1| - k_{22} - k_{11}(1 - p_1)}$ 100 percent of them are also received in \mathcal{D}_c . The process will be continue until the last cooperation round.

In the M rounds of conversation $\sum_{j=1}^{M/2} k_{2(2j)}$ packets will be received at D_1 . Out of these packets there are $\sum_{j=1}^{M/2} k_{2(2j)}(1 - \alpha_{2(2j)})$ packets that have not been received by D_1 through the sender. Therefore D_1 has $n(1 - p_1) + \sum_{j=1}^{M/2} k_{2(2j)}(1 - \alpha_{2(2j)})$ reliable packets and can perfectly decode the message sent by the sender provided that $nR < n(1 - p_1) + \sum_{j=1}^{M/2} k_{2(2j)}(1 - \alpha_{2(2j)})$ with

$$\alpha_{1i} = \frac{|\mathcal{D}_c| - \sum_{j=1}^{i-1} k_{1(2j-1)} \alpha_{1(2j-1)} - \sum_{j=1}^{i-1} k_{2(2j)} \alpha_{2(2j)}}{|\mathcal{D}_1| - \sum_{j=1}^{i-1} k_{1(2j-1)} - \sum_{j=1}^{i-1} k_{2(2j)} \alpha_{2(2j)}}$$

and

$$\alpha_{2i} = \frac{|\mathcal{D}_c| - \sum_{j=1}^i k_{1(2j-1)} \alpha_{1(2j-1)} - \sum_{j=1}^{i-2} k_{2(2j)} \alpha_{2(2j)}}{|\mathcal{D}_2| - \sum_{j=1}^i k_{1(2j-1)} - \sum_{j=1}^{i-2} k_{2(2j)}}$$

where

$$\begin{cases} \sum_{j=1}^{M/2} k_{1(2j-1)} \leq n(1 - p_1) \\ \sum_{j=1}^{M/2} k_{2(2j)} \leq n(1 - p_2) \\ \sum_{j=1}^{M/2} k_{1(2j-1)} + k_{2(2j)} \leq \beta_1 n(1 - p_{12}) + \beta_2 n(1 - p_{21}) \end{cases} \quad (2)$$

in which $\beta_1 + \beta_2 \leq 2$. With the same argument the message sent by the sender can be perfectly decoded at D_2 if $nR < n(1 - p_2) + \sum_{j=1}^{M/2} k_{1(2j-1)}(1 - \alpha_{1(2j-1)})$.

Insuring reliable decoding at receiver 1 and 2 the transmission rate can not exceed the following value $R = \min\{R_1, R_2\}$ with:

$$R_1 = (1 - p_1) + \sum_{j=1}^{M/2} \frac{k_{2(2j)}}{n} (1 - \alpha_{2(2j)})$$

$$R_2 = (1 - p_2) + \sum_{j=1}^{M/2} \frac{k_{1(2j-1)}}{n} (1 - \alpha_{1(2j-1)})$$

where k_{ij} , $i \in \{1, 2\}$ and $j \in \{1, M\}$, are selected as the values that maximize R .

Now let's assume a very specific case in which $(p_1, p_2, p_{12}, p_{21}) \in \mathcal{D}$ and $\beta_1 = \beta_2 = 1$. In this situation the constraints illustrated by (2) will be simplified to $\sum_{j=1}^{M/2} k_{1(2j-1)} + k_{2(2j)} = n(1 - p_{12}) + n(1 - p_{21})$. From these packets transmitted over cooperation channels, $n(1 - p_{12})$ is the number of packets sent from D_1 to D_2 and the rest are the packets transferred from D_2 to D_1 . They can be considered as two non-intersected subsets \mathcal{D}_{12} and \mathcal{D}_{21} of $\mathcal{D}_1 \cup \mathcal{D}_2$ with respective sizes of $n(1 - p_{12})$ and $n(1 - p_{21})$. Asymptotically with large M , $(1 - p_2)$ (resp. $(1 - p_1)$) percent of packets in \mathcal{D}_{12} (resp. \mathcal{D}_{21}) are also in \mathcal{D}_1 (resp. \mathcal{D}_2). Therefore $|\mathcal{D}_1 \cup \mathcal{D}_{21}| = 1 - p_1 p_{21}$ and $|\mathcal{D}_2 \cup \mathcal{D}_{12}| = 1 - p_2 p_{12}$ provided that $R = \min\{1 - p_1 p_{21}, 1 - p_2 p_{12}\}$.

The results show that for example in this specific situation the achievability rate can be improved using iterative coding schemes (for example if $p_2 p_{12} \leq p_1$ and $p_1 p_{21} \leq p_2$ the transmission rate achieved by this scheme is higher than the achievability region of coding schemes "a", "b", and "c"), but the cut-set bound is still non-achievable.

IV. CONCLUDING REMARKS AND EXTENSIONS

The capacity for the general discrete cooperative broadcast channel is not known which might seem to be not surprising since the general relay channel problem is not solved. As the capacity-achieving relaying scheme is known for the erasure relay channel we applied this scheme to the erasure CBC. It turns out that one does not achieve the capacity for all values of the erasure probabilities. More specifically the best coding scheme has not been determined in the case where $p_1 < p_{12}$ and $p_2 < p_{21}$. Of course, in practice, there will be many situations where the cooperation channels are relatively "good" and using the proposed cooperation scheme would be optimal. For example such a situation frequently occurs in densely populated cellular environments or in digital video broadcast systems where TV receivers are relatively close.

We also analyzed the benefits from using more decoding rounds ($M > 2$) to increase the transmission rate. We showed that in the case where $p_1 < p_{12}$ and $p_2 < p_{21}$, the achievable rate can be improved if we consider an infinite number of rounds of conversation. However, the transmission rate does not coincide with the traditional cut-set bound and therefore the capacity is not known. In this case, where the capacity could not be found with the proposed deterministic coding approaches, it would be relevant to adapt to the erasure CBC the scheme recently derived by [13] in the context of the relay channel.

We are currently working on derivation of a tighter converse bound for the capacity region of CBC erasure channels that is not derived from the general discrete cut-set bound but rather takes into account specificities of erasure channels. This approach has been followed by [10] to find an upper bound tighter than the conventional max-flow min-cut bound for the relay channel.

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